Path Planning and State Estimation for Unmanned Aerial Vehicles in Hostile Environments

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In this paper, planned paths for Unmanned Aerial Vehicles (UAVs) passing through the hostile environments with given starting and arriving points are generated and dynamic state variables for horizontal flight are estimated. The objective of the path planning is to maximize safety while maintaining a flyable path. The procedure starts from a Voronoi graph as initial coarse path. A reference path using parameterized Cornu-Spirals (CSs) is calculated to refine the initial Voronoi graph. The CS curve is represented by the polynomial expression of the path curvature. Hence, continuity and smoothness requirements for the designed flyable path are achieved by the continuous curvature characteristics of CSs. The refined path is generated by minimizing the sum of the orthogonal displacements from the Voronoi points, while satisfying the maximum curvature constraints, projection constraints and boundary constraints. A nonlinear programming solver is used to calculate the optimized parameters. As an example, the dynamic state variables of a UAV flying along the reference CS path at a constant altitude are estimated using an extended Kalman filter and smoother.

I. Introduction

Unmanned Aerial Vehicles (UAVs) are now widely used in anti-terrorism activities and intelligence gathering to enhance mission performance and maximize safety. The susceptibility of these UAVs in hostile environments raises requirements for flight path planning. Path planning strategies in hostile environments normally are comprised of two phases [1], [2]. The first phase is a Voronoi graph search, which will generate polygonal graphs and will

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optimize a safety performance index. The second is to use the virtual forces emanated from the virtual field of each surveillance radar site to refine the generated Voronoi graphs. These virtual forces provide information that can be used to reduce the vertices of the Voronoi polygonal and greatly improve the UAV performance. But, the curvature continuity of the refined graphs, which plays an important role in the stability of the UAVs turning maneuvers, often does not meet the requirements for a continuously flyable path.

Many kinds of curves have been studied and designed for UAVs to accomplish their mission [3]-[5]. Dubins curves, first applied in robotics path planning, are curves along which the UAV can move forward. This kind of circle-line-circle curve has a jump discontinuity in curvature at the connection points between the circle and the line that will cause a robot to stop at these connection points when traveling through the whole path. Other curves like those of Reeds and Shepp [6], [7] also have curvature discontinuities at their joint points. An alternative choice, the composite clothoid-line-clothoid curves, can be well designed with curvature being zero at the joint points to eliminate the discontinuities. This allows one to generate a curvature continuous path by using different kinds of simply shaped curves. While, under most circumstances, we are expecting more flexibility in the curve shape that will allow more space of change. Shanmugavel, et al. [3]-[5] have proposed quintic Pythagorean Hodograph (PH) curves for a flyable path with ten parameters representing each curve. The PH curves are flexible in design and their curvatures are expressed in continuous polynomials. The parameter calculation of a PH curve is an iterative process in order to satisfy different constraints. Such kind of curves can be further simplified with fewer parameters and a more efficient optimization algorithm. All the methods discussed leave room for improvement in the area of continuous curvature path planning.

The Cornu-Sprial (CS) [8], [9], also known as a clothoid or Euler's Spiral, has wide application in highway and railroad construction since it can be used to design gradual and smooth transitions in highway entrances or exits. Kelly and Nagy [10] used a parametric CS model to generate real-time, nonholonomic trajectories for robotics to minimize the terminal posture error. Here, we consider this CS model for use in UAV path planning and investigate how this parametric CS curve works under different constraints.

To generate a flyable and safe path with given starting and ending points for UAVs passing through areas covered at least partially by several radar sites, the path constraints considered here include: (1) minimum accumulative exposure to all radar sites, (2) continuous curvature throughout its length which will ensure flyable path, (3) maximum curvature corresponding to the maximum achievable lateral turn rate, (4) initial and final boundary
constraints. Unlike other path planning problems, including that of moving objects finding the final path that normally results in motion planning or trajectory planning with system dynamics [11], the path considered here is in a static object environment without dynamic constraints. The work in this paper is based on the developed Voronoi graph; it refined the graph by proposing a generalized CS curve along with simplified parameter identification procedure.

Most of papers on the topic of path planning do not include the information about dynamic state variables of UAVs flying along the planned path. For control purposes, it is beneficial to estimate these state variables to construct entire information of the flight. Kalman Filtering [12] has been widely used as an efficient tool in optimal filtering and prediction, especially in the field of state estimation of UAVs performing designated missions [13-17]. For example, Grillo and Vitrano used an Extended Kalman Filter (EKF) to estimate the state variables and wind velocity for a nonlinear UAV model with Global Positioning System (GPS) measurements. Abdelkrim, et al. used an EKF and an $H_\infty$ filter to estimate the localization of UAVs whose position, velocity and attitude are measured by an Inertial Navigation System. Campbell, et al. used two different estimators to provide online state and parameter estimation for path planning in uncertain environments. The state estimation in these studies have a commonality because in both cases the UAVs have sensors or other instruments to provide useful measurement information. If all of the Voronoi points on the initial path are fixed and expected to be followed as close as possible, they can be assumed as measurement points. The generated CS curve is then treated as reference solution, so that the state variables can be estimated by the EKF.

The following sections present the procedure for the path information construction in three parts. The first part is the initial rough path of the Voronoi graph and the dynamic programming search algorithm. In the second part, CS curve expression and properties are introduced and different constraints and their mathematical expression are explained. This is followed by the systematic solution, nonlinear programming (NLP) solver. In the last part, the state variables are estimated based on the generated Voronoi points and the refined reference path generated in the first two parts. Simulation results are presented in each part separately.

II. Initial Path Generation

There are many developed graph search methods that maybe used to generate the initial path that will maximize the safety of the UAVs. Two prominent ones are heuristic search [18] and probability map search [19]. The Voronoi
graph search is generally used to find the rough initial path in the form of connected edges. The authors of some articles have stated the Voronoi method and the following dynamic programming search algorithm in detail [1, 2]. Here, we briefly describe them.

**A. Voronoi Graph Search**

A Voronoi diagram will decompose a space defined by random scattered points into separated cells in such a manner that each cell will include one point that is closer to all elements in this cell than any other points. The graph is constructed by using Delaunay triangulation and its dual graph, Voronoi diagrams. The procedure of this cell decomposition starts with a priori knowledge of the location and of number of the scattered points assumed as radar sites here. As illustrated in Fig. 1, the star points are pre-defined radar sites. The circle points named as Voronoi points are the centers of the circumcircle of the triangles formed by three of the Voronoi sites without including any other sites in this circumcircle. The formation of these triangles is called Delaunay triangulation. All points on edges connected by the Voronoi points have equivalent distance to two adjacent Radar sites. By connecting all of the edges together, polygons are formed and these polygons construct Voronoi graph. The UAV starting point and ending point are also illustrated as small triangles in the plot.

If UAVs are assumed to go through the radar coverage area along the edges in the formed Voronoi Graph, the probability of exposure of each edge to radar sites can be determined using the distance from the points on the edge to the radar sites. Suppose each radar site emits a radiation signal with equivalent strength in all directions, the threats of exposure to this radar site is proportional to the inverse of distance to the forth power. The whole edge threat cost is the accumulative threat calculation from the beginning of the edge to the end. Mathematically, it is expressed as [1]

\[
\int_{x_1}^{x_2} \frac{\sqrt{dx^2 + dy^2}}{\left[ (p_x - x)^2 + (p_y - y)^2 \right]^2} dx
\]  

(1)
where the pair \((x_1, y_1)\) and \((x_2, y_2)\) are the coordinates of starting and ending points on the edge and \((p_x, p_y)\) is the radar site coordinate. Since the edge position can be expressed by the starting and ending points as

\[ y = y_1 + \alpha(x - x_1), \quad \alpha = \frac{y_2 - y_1}{x_2 - x_1} \]  \hspace{1cm} (2)

so that Eq. (1) can be simplified as

\[
\int_{x_1}^{x_2} \sqrt{1 + \alpha^2} \left((p_x - x)^2 + (p_y - y_1 - \alpha(x - x_1))^2\right)^{\frac{1}{2}} \, dx
\]  \hspace{1cm} (3)

By assigning a threat cost to each edge, an optimal path with lowest probability of detection when passing through the radar area can be constructed. Determining this optimal path requires the calculation of the threat cost for all of the available paths and then selects the lowest cost one after comparison. The dynamic programming method is applied here to solve these kinds of optimal path selection problem.

**B. Dynamic Programming**

Dynamic Programming (DP) is a branch of Operational Research first proposed by Richard Bellman. The DP procedure is to break a problem into many layers of subproblems and then solve each subproblem individually to achieve the overall optimality. At each layer, the optimal solution is saved so that when we come to this layer again, we can retrieve the saved result to avoid repeated computing. In the Voronoi graph of Fig. 1, the cost of each edge is determined and the objective is to find a path from the starting point to the ending point with lowest total cost. Counting from the ending point backward, at each Voronoi point, the lowest cost from that point to the ending is computed and saved. For example, Voronoi point 5 is set with a cost of zero and the cost from Voronoi point 4 to 5 is \(8.773 \times 10^{-5}\). This process is repeated for all Voronoi points. When a point is reached that has a cost value already determined, the algorithm takes the pre-defined optimal path from that point to the ending without searching them again. So when counting the cost at point 2, it will take the determined route from 3 to the ending point. Finally, the safest path is found and labeled with dark lines in Fig. 2.
III. Reference Path Generation

Obviously, the initial path generated in Fig. 2 is coarse and not flyable by UAVs. The final flight path is expected to be smooth enough everywhere so that the UAVs won’t have sharp turns on the whole path. At the same time, we also expect the path to be as close as possible to the original Voronoi graph to maximize the safety factor. Since the curvature is a key factor in determining the smoothness of a curve, the problem turns out to be a curve fitting problem with curvature constraints and some other constraints. The CS with advantages introduced in the following is selected as the reference path to fit the Voronoi points and satisfy the specified constraints.

A. Cornu-Spiral

The CS is a well known curve whose curvature is defined as a polynomial function of its arc length \( s \) as

\[
\kappa(s) = \sum_{i=0}^{n} \alpha_i s^i
\]

and the curvature is also the derivative of the curve angle \( \theta \) with respect to the arc length:

\[
\kappa(s) = \frac{d\theta}{ds}
\]

The positions of the points on this curve in Cartesian coordinates are calculated by the "Fresnel Integrals":

\[
x(s) = \int_0^s \cos(\theta(s)) ds \\
y(s) = \int_0^s \sin(\theta(s)) ds
\]

One advantage of using this curve in solving the path planning problem is the simplicity of calculation of its arc length:

\[
s(t) = \int_0^t \sqrt{\dot{x}(t)^2 + \dot{y}(t)^2} dt = \int_0^t \kappa dt = t
\]

The above expressions are general, i.e., they apply to all forms of CS curves including special cases like a straight line (\( n = 0, \alpha_0 = 0 \)), circle (\( n = 1, \alpha_1 = 0 \)) and a unit CS (\( n = 1, \alpha_0 = 0, \alpha_1 = 1 \)). In a unit CS, the curvature equals the arc length and will therefore increase with the length. So the longer the curve, the greater the curvature. Hence, the origin of the term “spiral”. The coefficients \( \alpha_i \) affect the increase rate of the CS curve. In Fig. 3, all the CS curve curvature polynomials are order \( n = 1 \) with \( \alpha_0 = 0, \alpha_1 \) varied from 0.1 to 5. It is obvious
that when transversing the same arc length, the larger the value of $\alpha$, the more the curvature increases. In addition, the order in the curvature expression also affects its shape. Fig. 4 shows a series of CS curves with different order numbers in the curvature polynomials. The coefficients of the highest order are all for $\alpha_n = 1$ with the others set as zero. It can be seen that curves with the higher order polynomials “curl” faster than others.

Combining the two types of effects that determine the shape of the CSs, we can produce an expression of the curvature polynomials to define any CSs possessing as many degrees of freedom parameters as necessary to meet the required constraints. In the problem formulation stated above, the primary constraint for such curves is to meet the initial and final conditions which are classified as equality constraints. If the initial conditions $P_0 = (x_0, y_0)$ are incorporated as parameters in the CS curve, at least two additional parameters are required to meet the final boundary constraints.

To have enough freedom to satisfy other inequality constraints and coordinate all the parameters to achieve the optimization purpose, one more parameter is required. Although the inclusion of more parameters in the CS expression will increase the flexibility of the curve, it will also increase more calculation burden. As a compromise between the optimality and computation time, five parameters are used in one CS curve finally. Three of them, $a$, $b$ and $c$, form the cubic polynomials of the curve angle expression. $\theta_0$ is the initial heading angle. The last one is the final arc length $s_f$. These parameters enter the problem formulation as follows:
\[
\theta(s) = \theta_0 + as + bs^2 + cs^3
\]
\[
\kappa(s) = \dot{\theta}(s) = a + 2bs + 3cs^2
\]
\[
x(s) = x_0 + \int_0^s \cos(\theta_0 + as + bs^2 + cs^3)ds
\]
\[
y(s) = y_0 + \int_0^s \sin(\theta_0 + as + bs^2 + cs^3)ds
\]

Then, the path planning problem has been cast as a parameter optimization problem with equality and inequality constraints.

B. Continuous Curvature Constraints

Continuous curvature is an important factor in designing a flyable path, because physically the vehicle’s path must be continuous and the rate of change of curvature with arc length is related to the directional command, a key input in maneuvering UAVs. Discontinuities in the curvature will cause jumps in the steering angle input. Such discontinuous input is not achievable. The curvature of a planar curve defined by \(x(t)\) and \(y(t)\) is

\[
\kappa(t) = \frac{\dot{x}(t)\ddot{y}(t) - \dot{y}(t)\ddot{x}(t)}{(\dot{x}(t)^2 + \dot{y}(t)^2)^{3/2}}
\]

The CS curve specified in (8) with three parameters representing its polynomials can be expressed as

\[
\kappa(s) = a + 2bs + 3cs^2
\]

Physically, the first and second derivatives of curvature correspond to the lateral velocity and acceleration of a point (the vehicle) that is moving along the curve. Therefore, the curvature must be at least twice differentiable to meet the continuous velocity and acceleration requirement. The CS curve with curvature expression in (10) has second-order polynomials, is twice differential and can satisfy the continuity constraints.

C. Maximum Curvature Constraints

In CS expression of (8), the curvature is the only shape determining factor. By using the maximum curvature constraint, the kinematic acceleration of UAVs can be limited. If a proper tangential velocity is maintained then the path designed can actually be traversed. Since the curvature is a parabolic function of the arc length, its maximum value points can only occur at one of three points: the initial point, the summit or bottom point, or the final point. Mathematically, the curvature values at those three points are
\[ \kappa_0 = a \]
\[ \kappa_m = a - \frac{b^2}{3c} \]
\[ \kappa_f = a + 2bs_f + 3cs_f^2 \]  

(11)

If the curvature at none of these three points exceeds the maximum value, then the curvature throughout the whole length will be suitably constrained.

D. Voronoi Points Projection Constraints

In order to fit the Voronoi points, it’s necessary to minimize the distances from the Voronoi points to their projection points on the CS curve. Before the parameters of the CS curve are defined, it is difficult to calculate the orthogonal distance, but it is obvious to see that the slope of the normal line for the projection point on the CS is the negative reciprocal of its tangent line slope. Assume there are \( n \) Voronoi points. The projection of Voronoi point \( i \) \((= 1, \ldots, n)\) on the CS curve will satisfy the following relationship

\[
\frac{p_{yi} - y(s_i)}{p_{xi} - x(s_i)} \cdot \tan(\theta(s_i)) = -1
\]

(12)

where \((p_{xi}, p_{yi})\) are the coordinates for the Voronoi point \( i \) and \( s_i \) is the length of the CS from the starting to the corresponding projection point with coordinates of \((x(s_i), y(s_i))\). Instead of calculating the projection points by geometric theory, they can be found by putting the constraints on \( s_i \) into Eq. (12). Then, the distance from Voronoi point \( i \) to its corresponding projection point is calculated as

\[
d_i = \sqrt{(p_{xi} - x(s_i))^2 + (p_{yi} - y(s_i))^2}
\]

(13)

E. Boundary Condition Constraints

The initial and final conditions are the equality constraints on the system. The path designed by the CS curve starts from the original points, so the initial condition is satisfied by setting the initial position coordinates as parameters in the CS curve

\[
x(0) = x_0 \\
y(0) = y_0
\]

(14)
The final point is determined by using a Runge-Kutta numerical integration method to satisfy the specified final boundary conditions.

At this point, all the constraints have been described. Some of them can be satisfied by the properties of the CS curve itself, others are expressed in corresponding equalities and inequalities. So the path planning problem is now a parameter optimization problem in which the parameters of the CS curvature expressions are identified while minimizing the accumulative orthogonal displacements for all Voronoi points.

**F. Nonlinear Programming (NLP) Solver: SNOPT 6.2**

The NLP solver used to solve the NLPP of interest here is SNOPT [20], which is based on a Sequential Quadrature Programming (SQP) algorithm and has been used as efficient tool to solve nonlinear optimal control problems with constraints [21-24]. SNOPT can be used to solve problems like the following:

Minimize a performance index \( J(x) \), subject to constraints on individual state and/or control variables:

\[
x_L < x < x_U
\]

constraints defined by linear combinations of state and/or control variables:

\[
b_L < Ax < b_U
\]

and/or constraints defined by nonlinear functions of state and/or control variables:

\[
c_L < c(x) < c_U
\]

With the above in mind, we can transfer the path planning problem into a simple form.

Let the NLP performance index \( J \) by:

\[
J = \sum_{i=1}^{n} \left[ (p_{x_i} - x(s_i))^2 + (p_{y_i} - y(s_i))^2 \right]
\]

and the NLP variables of the CS curve as:

\[
x = [\theta_0, a, b, c, s_1, \ldots, s_i, \ldots, s_n, s_f]
\]

where \( i(=1, \ldots, n) \), subject to the equality constraints including the Voronoi points projection constraints and boundary conditions, \( P_0 \) and \( P_f \). These constraints will ensure the curve fitting will minimize the orthogonal
displacement and all the coordinates on the CS curve will satisfy its differential function in trigonometric form of Eq. (8). In addition, the inequality constraints to ensure the flyable path can be set as:

\[
\begin{align*}
-|\kappa_{\text{max}}| & \leq a \leq \kappa_{\text{max}} \\
-|\kappa_{\text{max}}| & \leq a - \frac{b^2}{3c} \leq \kappa_{\text{max}} \quad \text{if } \left( s_0 \leq -\frac{b}{3c} \leq s_f \right) \\
-|\kappa_{\text{max}}| & \leq a + 2bs_f + 3cs_f^2 \leq \kappa_{\text{max}}
\end{align*}
\]  

(20)

For the Voronoi sets in Fig. 2, a CS curve to fit those points is calculated under two conditions, one without maximum curvature constraint and the other one with maximum curvature constraint $|\kappa_{\text{max}}| = 1/3$. Their path simulation results are illustrated in Fig. 5 and Fig. 6, respectively. The corresponding curvature results with respect to curve length are illustrated in Fig. 7 and Fig. 8, respectively. From the two kinds of simulation results, the CS curve without curvature constraint fits the Voronoi points better than the one with curvature constraint by increasing the lateral kinematic acceleration to follow the path. The accumulative threat of the CS curve to the radar sites are 0.495 and 0.512 for curves without and with curvature constraint, respectively. Compared to the accumulative threat of the Voronoi graph, 0.439, the CS curve achieved a flyable path with similar level of threat exposure to the radar sites.
IV. State Estimation

A. UAV Model

For the simplicity of illustration, a two-dimensional UAV nonlinear model [25] in horizontal flight is used here. Assuming all the adversarial radar sites are located in the same altitude so that the generated Voronoi graph and reference path are in the lateral plane. The three degree of freedom UAV equations of motion are listed below. This model is based on the assumption that the UAV is a rigid body flying over a non-rotating and flat Earth.

\[
\begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{\psi} \\
\dot{r}
\end{bmatrix} = 
\begin{bmatrix}
\ddot{u} - F_x / m \\
\ddot{v} + F_y / m \\
\ddot{\psi} = r \\
\dddot{r} = N / I_{zz}
\end{bmatrix}
\]

(21)

where \((x, y)\) is the position, \((u, v)\) is the lateral velocity components, \(\psi\) is the heading angle and \(r\) is the yaw rate. \([F_x, F_y, N]\) are the aerodynamic forces and moment achieved by the control actuators. \(m\) is the mass and \(I_{zz}\) is moment of inertia about the \(z\) axis of the UAV body frame. This UAV model is built in time domain, but the position and heading angle variables of the reference path are built in distance domain and their derivatives with respect to distance are
\[
\begin{align*}
    x'(s) &= \cos(\theta(s)) \\
    y'(s) &= \sin(\theta(s)) \\
    \theta'(s) &= \kappa(s)
\end{align*}
\] (22)

In order to estimate all variables in the time domain, it is necessary to change all the variables of the reference path from distance domain to time domain by calling

\[ ds = V(t)dt \] (23)

where \( V(t) \) is the time history of the velocity magnitude. By substituting \( ds \) into Eq. (21), the equations of motion in time domain are expressed as

\[
\begin{align*}
    \dot{x}(t) &= V(t)\cos(\theta(t)) \\
    \dot{y}(t) &= V(t)\sin(\theta(t)) \\
    \dot{\theta}(t) &= V(t)\kappa(t)
\end{align*}
\] (24)

By this transformation, all the state variables of the UAV model can be calculated according to the generated reference path, which will then provide reference solution for linearization of the dynamic model in EKF.

**B. Design of Extended Kalman Filter and Smoother (EKFS) and Simulation Results**

An EKFS [12] can often make good estimates of the state variables when the measurements are under random disturbance. By solving the weighted least squares problem for the measurements, we can determine the most likely values of the state variables for continuous or discrete systems. The procedure of EKF consists of two parts: a prediction part to predict the current states from the last step and an updating part will add a correction term to the predicted states to refine them and get more accurate values. The optimal smoother is a backward sweep procedure starting from the last time point back to the first. It will take advantage of the filtered estimates in the forward EKF and calculate the differences between them and the final smoothed values. The optimal smoothing is also a recursive computation to further refine the estimates.

Due to the precision limitation of the detection instrument, the adversarial radar sites location detected is under some degree of uncertainty, which causes the subsequent uncertainties of the Voronoi Points searched in the initial graph. By implementation the above mentioned EKFS, the errors during the measurement can be reduced and the adjustments of the state variables are estimated and added to the nominal solution obtained in the EFKS algorithm.
Since the UAV model is nonlinear, we need to derive a Jacobian matrix $A$ by taking the derivatives of the nonlinear system equations with respect to the state variables. Its mathematical form is written as

$$A = \begin{bmatrix}
0 & 0 & \cos\psi & -\sin\psi & -u\sin\psi - v\cos\psi & 0 \\
0 & 0 & \sin\psi & \cos\psi & u\cos\psi - v\sin\psi & 0 \\
0 & 0 & 0 & r & 0 & v \\
0 & 0 & -r & 0 & 0 & -u \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$  \hspace{1cm} (25)

Similarly, a Jacobian matrix $B$ derived by making the derivatives of the nonlinear system equations with respect to the control variables is

$$B = \begin{bmatrix}
0 & 0 & 1/m & 0 & 0 & 0 \\
0 & 0 & 0 & 1/m & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1/I_z
\end{bmatrix}^T$$  \hspace{1cm} (26)

According to the observation output, the observation matrix $H$ is simply defined as

$$H = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0
\end{bmatrix}$$  \hspace{1cm} (27)

These Jacobian matrices, estimated at the nominal state points $X_n$, are then used in the error approximation function expressed as

$$\Delta\dot{X} = \dot{X} - \dot{X}_n = A\Delta X + B\Delta U + H.O.T$$  \hspace{1cm} (28)

Then, the EKFS will find an optimized adjustment $\Delta\hat{X}$ to the nominal solution by going through its filter and smoother. Design of the EKFS also includes the selection of the covariance matrices of the disturbance and observation error, $Q$ and $R$, respectively. If the measurement noise is carried out by the characteristics of the measurement instrument only, the elements of $R$ can be determined from the stochastic properties of the measuring system. Noting that the initial and final position are fixed, $R$ at those points are set as extremely small numbers. The elements in $Q$ and $R$ directly affect the Kalman gain $K$, thereafter the adjustments require considerable effort to find the best fit. In this problem, $Q$ is a one by one identity matrix and $R$ is set as 4 time a two by two
identity matrix except the starting and ending points where $R$ is 0.4 times a two by two identity matrix. We also assume that $w$ and $v$ are unrelated with zero mean values.

Simulations were carried out based on two kinds of reference paths generated above, one with and the other without maximum curvature constraints. The speed of the reference paths was assumed to be constant cruise speed at
maximum weight. Calculated measurements are the position of Voronoi points labeled in Fig. 1. The starting and ending points are fixed measurements with zero errors. The state variable history of the reference solution and estimated solution are plotted in Fig. 9 and Fig. 10 for these two cases, respectively. By adding a covariance error to the reference solution, the estimated values further reduce the measurement errors.

V. Conclusions

A type of UAV paths with continuous curvature and constrained turning acceleration has been designed using the Voronoi graph and parameterized Cornu-Spirals (CS). The state variables for such kind of planning path are estimated by the Extended Kalman Filter and Smoother (EKFS). The planned paths are the sub-optimal solution of the Voronoi diagram which will maximize the safety factor. They refined the initial path by using the continuous curvature characteristics of the CSs and try to fit the Voronoi points with the minimum orthogonal distance. The curve fitting algorithm includes all parameters of CS as nonlinear programming variables in the nonlinear programming solver and gets the solution in one effort without tedious and iterative testing to adjust the coupled variables. This paper also considered disturbances in the detection and acting of the hostile radar sites and estimated the state variables based on the reference path generated forehead. Finally, the three parts of this paper: initial coarse graph, refined reference path and state estimation can be used to build up a complete information data set for both path planning and control purposes. Multiple UAV paths which require cooperative control to increase simultaneous working performance will be considered in the future work.

References


